Technical Comments

Comment on "Wind Shear Terms in the Equations of Aircraft Motion"

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IN Ref. 1, Frost and Bowles have discussed the inclusion of variable winds in the equations of motion of airplanes. Their treatment of the relative motion of the atmosphere is unfortunately incomplete and leads to wrong answers for the aerodynamic moments associated with the wind gradient. This can be clearly appreciated by considering the rolling moment term. The extra moment associated with the roll component of wind gradient, according to Eq. (31) in Ref. 1, is

$$\Delta L = -L_p \frac{1}{2}\left(\frac{\partial W_z}{\partial y} - \frac{\partial W_y}{\partial z}\right)_B \tag{1}$$

[The ½ is inadvertently omitted in Ref. 1 between Eqs. (30) and (31).]

Figure 1 shows the velocity fields associated with the two gradients $\partial W_z/\partial y$ and $\partial W_y/\partial z$. It is clear that, whereas a wing that lies approximately in the *x-y* plane is strongly affected by $\partial W_z/\partial y$, it is virtually unaffected by $\partial W_y/\partial z$! Hence, to simply take the difference between these two as the driving term for rolling moment is clearly wrong. The correct additional term is, in fact, simply

$$\Delta L = -L_p \frac{\partial W_z}{\partial y} \tag{2}$$

The reason for this is that $(\omega - \omega_w)$ as used in Ref. 1 does not describe all of the relative motion that is important. In addition to rotation, there is a *shearing* deformation, which when included in the analysis leads to Eq. (2) instead of Eq. (1). This point is discussed in Ref. 2 and the earlier work cited therein. The result for a linear-wind model, as postulated by Frost and Bowles, is that the wind gradients lead to only four significant inputs for airplanes, denoted in Ref. 2 as $\{p_g q_g r_{1g} r_{2g}\}$. These result in additional rolling, pitching, and yawing moments on the airplane. Some computation time is saved by calculating only four of the total of nine possible gradients.

When the calculations have to be made in real time, as in flight simulators, computational efficiency is very important if high-fidelity simulation is to be achieved at a rapid enough update rate. Some improvements in speed can be obtained with a formulation different from the two options presented in Ref. 1. Instead of calculating Euler angles by Eq. (21) and using them to compute the matrices for L and $\dot{L}$ via Eqs. (5) and (3) in Ref. 1, one can use Eq. (5.2,11) of Ref. 3, which yields

$$\dot{L}_{BE} = -\tilde{\omega} L_{BE} \tag{3}$$

where

$$\tilde{\omega} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}$$

The use of Eq. (3) avoids the time-consuming trigonometric functions that appear in Ref. 1 in Eqs. (5), (13), and (21). Denoting $L_{BE} = [\ell_1 \ell_2 \ell_3] = [\ell_{ij}]$, in which ℓ_k are the orthogonal unit vectors of frame F_E, Eq. (3) is equivalent to nine simple differential equations, the first of which is

$$\dot{\ell}_{11} = -r\ell_{21} + q\ell_{31} \tag{5}$$

Not all nine equations need be solved, however. For example, if one solves first for ℓ_1 and ℓ_2, using six of the differential equations, ℓ_3 is then obtained from the simple algebraic relation

$$\ell_3 = \ell \times \ell_2 \tag{6}$$

If the Euler angles are needed for input to the simulator drive, they can be updated as frequently as needed from

$$\theta = -\sin^{-1}\ell_{13}$$

$$\phi = \tan^{-1}(\ell_{23}/\ell_{33})$$

$$\psi = \tan^{-1}(\ell_{12}/\ell_{11}) \tag{7}$$

Finally, with the quasistatic aerodynamic model appropriate to simulation, the only rate variable needed is $\dot{\alpha}$ (for use with $C_{L_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$), and this can be adequately represented by $\dot{\alpha} = \dot{w}/V$. Thus, the statement in Ref. 1 that all three of $[\dot{u}\dot{v}\dot{w}]$ are needed, regardless of whether the force equation is solved in F_B or F_E, is reduced to a need for only $\dot{w}$ if F_E is used. It may, therefore, be more efficient to solve the force equations in F_E, not in F_B. This avoids calculating $\dot{u}$ and $\dot{v}$ in each time step. The value of $\dot{w}$ needed is obtained essentially as outlined in Ref. 1 for the inertial-frame option, but using the simple form of $\dot{L}$ given in Eq. (3) above.

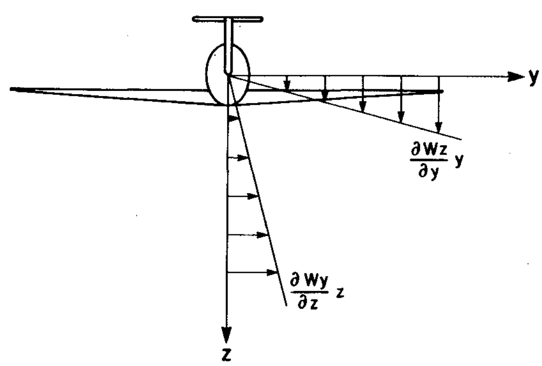

Fig. 1 Velocity fields.

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Comment on "Body of Revolution Comparisons for Axial- and Surface-Singularity Distributions"

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IN a recent Note,[1] D'Sa and Dalton present two comparisons of numerical potential flow surface velocity results for a surface source and an axial source singularity method. This work presents useful comparisons between the two techniques and points out the likelihood that axial-singularity methods may yield inaccurate solutions with arbitrary choices of size distribution of the axial-singularity elements. This dependence of solution accuracy on the size distribution of axial-singularity elements appears to be stronger for the higher-order formulations (e.g., quadratic), as seen in Fig. 2 of Ref. 1. Certainly, the general conclusion that surface-singularity methods are as a result more reliable than axial-singularity methods is valid. However, some limitations to this work must be pointed out. These limitations occur in two general areas: the limited usefulness of the ellipsoidal body as a test case for axial-singularity methods, as recently pointed out by Hess,[2] and what appears to be an inadequate, or less than optimal, formulation for the axial-singularity method, especially as applied to the second test geometry of Ref. 1.

First, as Hess reminds us in Ref. 2, the axisymmetric potential flow past an ellipsoid may be *exactly* represented by a line source distribution of linearly varying intensity, located between the foci, independent of body slenderness ratio. Hence, it is most likely that axial line source singularity numerical results for assumed piecewise linear source intensity will be a measure of the influence of finite word length and accumulated truncation error in the calculation, assuming that the proper inset of the axial source distribution is used.

Also, based on the presented results for the axial source distributions for the ellipsoid (Fig. 1, Ref. 1), it appears likely that these calculated results were obtained for a nonoptimal inset of the axial source distribution. Similar small oscillating speed or pressure coefficient error distributions were seen by Kuhlman and Shu[3,4] for ellipsoids at zero angle of attack, using their axial-singularity formulation if nonoptimal inset was used. As found empirically,[3,4] and as shown by Moran,[5] this optimal inset distance is the distance from the near focus to the nose of the ellipsoid. Using the optimal inset, calculated errors in surface pressure coefficient for ellipsoids were found in Ref. 3 to be $0(10^{-5})$ using 60-bit word length, a flow tangency boundary condition formulation, and 15 linearly varying line source elements in a cosine size distribution to represent each half of the body.

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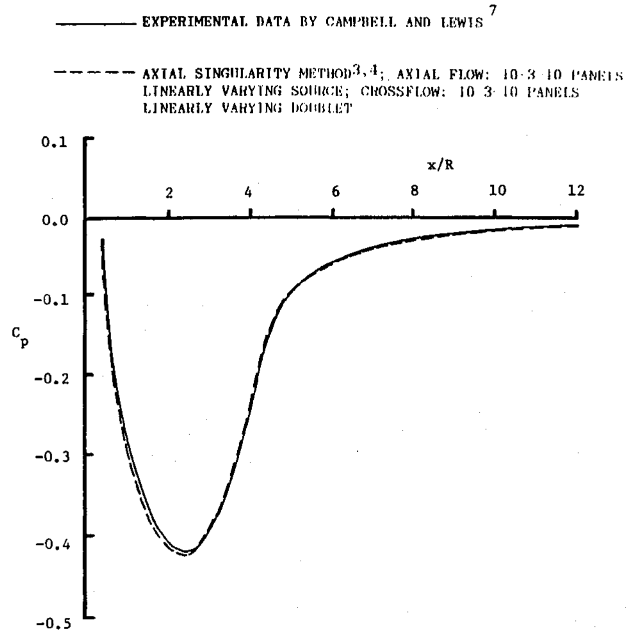

Fig. 1 Pressure coefficient distribution along top meridian line for inclined flow; $\alpha = 6.08$ deg past a cylindrical body with $SR = 2$ ellipsoidal nose, using axial-singularity method, compared with experiment.

Further, using this inset for a slenderness ratio 5 ellipsoid at zero angle of attack, results presented in Ref. 3 showed a continuous trend of decreasing rms error as the number of singularity elements was varied from 10 to 40 for either constant or linearly varying axial source or doublet singularities, using either equal-sized singularity elements or a cosine-type size distribution. This consistent decrease in rms error in calculated surface pressure coefficient distribution contrasts with results presented by D'Sa and Dalton, in which surface velocity error was observed to begin to increase for the ellipsoid test case when more than 35 linear source elements were used. This must be the result of differences in accumulated errors due to finite word length, where the results of Refs. 3 and 4 were obtained using single-precision calculations on CDC hardware, which is roughly equivalent to using double-precision arithmetic on many other computers.

The work of Kuhlman and Shu[3,4] documented the extension of axial-singularity methods to potential flows past axisymmetric bodies at nonzero angle of attack, as suggested by Karamcheti.[6] Accuracies at nonzero angle of attack were identical to those at zero angle of attack, and accuracy was independent of slenderness ratio. It appears likely that a superposition of the piecewise linearly varying cross-flow line-doublet singularity distribution of Kuhlman and Shu[3,4] with the axisymmetric surface-singularity method described by D'Sa and Dalton in Ref. 1 would be a very powerful method for calculation of potential flow at nonzero angle of attack past an arbitrary axisymmetric body.

The second limitation to the comparisons presented in Ref. 1 lies in what must be an inadequate formulation of the higher-order (linear and parabolic) axial source methods utilized for the more complex body shape shown in Fig. 2 of Ref. 1. This body shape has a flat midsection and, hence, a discontinuity in curvature, from a finite value to zero, as one moves from the nose section to the cylindrical midsection or from the midsection to the tail. As shown in Refs. 3 and 4, accurate representation of the potential flow around such bodies is possible at least for the linear source distribution, both at zero and nonzero angle of attack, only if the usual continuity-of-source-strength requirement is eliminated for the two abutting axial-singularity elements at the nose-cylinder or cylinder-tail juncture. This source strength con-